International Journal of Engineering and Technology Research
Vol. 1, No. 7, August 2013, PP: 116 - 126, ISSN: 2327-0349 (Online)
Available online at www.ijeatr.org

Research article

Analysis of Magnetic and Injection Effects on Unsteady Heat and Mass Laminar Flow past a Vertical Surface

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Abstract

This study is devoted to investigate the magnetic and injection effects on unsteady heat and mass laminar flow past a vertical surface under a convective surface boundary condition. The unsteadiness is due to the time dependent velocity. Using a similarity variable, the governing nonlinear partial differential equations have been transformed into a set of coupled nonlinear ordinary differential equations. The resulting equations are solved numerically using Nachtsheim-Swigert shooting iteration technique along with Runge-Kutta initial value solver. The fluid-solid interface characteristics for different values of the thermo-physical parameters, which characterizes our convection processes were examined on the velocity, temperature and concentration profiles. Numerical data for the skin-friction coefficients, Nusselt and Sherwood numbers are tabulated, shown graphically and discussed.

Keywords: Laminar flow, heat and mass transfer, magnetic and injection effect, unsteady.

Introduction

The phenomenon of free convective flow with simultaneous heat and mass transfer has been a subject of interest of many researchers because of its varied applications in natural sciences, engineering sciences and in industry. Such phenomenon is observed in buoyancy induced motions in the atmosphere, in bodies of water, quasi-solid bodies such as earth, etc.

Investigations of unsteady heat and mass laminar flow of viscous fluids over a vertical surface are important in many manufacturing processes, such as polymer extrusion, drawing of copper wires, continuous stretching of plastic films and artificial fibers, hot rolling, wire drawing, glass-fiber, metal extrusion, and metal spinning.

The influence of magnetic field on viscous incompressible fluid of electrically conducting is of importance in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, textile industry etc. In many process industries the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads etc., in the presence of an electrically conducting fluid subjected to magnetic field. (Palani & Srikanth, 2009)

Chamkha (2002) investigated the laminar hydromagnetic natural convection flow along a heated vertical surface in a stratified environment with internal heat absorption. Shapiro and Fedorovich (2004) obtained the exact solution for the start-up and transition to steady state for the natural convection flow adjacent to an infinite vertical plate with stratified ambient using a Laplace transform technique that could only be applied at $Pr < 1$.

Borkakati and Chakraborty (2002) investigated the nature and behaviour of a viscous, incompressible, electrically conducting fluid over a flat plate which is moving with a uniform speed in a quiescent fluid in presence of a uniform magnetic field. In their conclusion they have found that for an incompressible fluid, both the fluid velocity and temperature gradually decreases with the increase of viscosity parameter. Elbashbeshy (1996) studied heat and mass transfer in the same problem in presence of variable transverse magnetic field.

Das (2009) investigated the effect of suction and injection on MHD three dimensional couette flow and heat transfer through a porous medium

Loganathan et. al.(2010) analysis the effects of thermal conductivity on unsteady MHD free convective flow over a semi-infinite vertical plate. Soundalgekar and Ganesan (1981) investigated the finite difference of transient free convection with mass transfer of an isothermal vertical flat plate. Mahanti and Gaur (2009) presented the effects of varying viscosity and thermal conductivity on steady free convective flow and heat transfer along an isothermal vertical plate in the presence of heat sink.

Thus, the aim of this paper is to study an unsteady two dimensional free convective heat and mass laminar flow past a vertical surface under a convective surface boundary condition, with magnetic and injection effect. Hence, numerical calculations were carried out to investigate the effects of magnetic field parameter (M), unsteadiness parameter (A), thermal Grashof number (Gr), concentration Grashof number (Gc), injection parameter (f_0), Prandtl number (Pr) and Schmindt number(S_C), on velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number of such the flow. Results are presented in the table and illustrated graphically.

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Mathematical Formulation

Consider unsteady two-dimensional laminar flow of a viscous incompressible fluid in the presence of transverse magnetic field of constant intensity and injection. The x-axis is taken along the vertical surface in the upward direction and the y-axis normal to the surface. The fluid is assumed to flowing over the surface with a velocity U(x,t) along the x-axis and y-axis is normal to it. Under these assumptions the Boussinesq's approximation for the flow field is the governing equations of continuity, momentum, energy and diffusion under the influence of uniform transverse magnetic field and injection is given by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta_T(T-T_\infty)+g\beta_C(C-C\infty)-\frac{\sigma\beta_o{}^2}{\rho}u \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

With the following the Boundary conditions,

$u=U_w, v=V_w, T=T_w, C=C_w$, at $y=0$

$u\to 0, T\to T_\infty, C\to C_\infty$, as $y\to\infty$ for $t>0$

We now introduce the Stream function $\psi = x\sqrt{\frac{c\upsilon}{1-\lambda t}}f(\eta)$ as defined by:

$$u=\frac{\partial\psi}{\partial y} \quad \text{and} \quad v=-\frac{\partial\psi}{\partial x}$$

and the similarity variable

$$\eta = y\sqrt{\frac{c}{\upsilon(1-\lambda t)}} \tag{5}$$

$$\text{Also, } T=T_\infty+T_o\left[\frac{cx}{2\upsilon(1-\lambda t)^2}\right]\theta(\eta) \tag{6}$$

And $C = C_\infty + C_o \left[\dfrac{cx}{2\upsilon(1-\lambda t)^2} \right] \phi(\eta)$ (7)

Substituting (5), (6) and (7), the dimensionless quantities, into the governing equations, the transformed Equations (1), (2), (3), and (4) with the transformed boundary conditions are:

$$f'''(\eta) + f(\eta) f''(\eta) - [f'(\eta)]^2 - \frac{A}{2}\eta f''(\eta) + G_T \theta(\eta) + G_C \phi(\eta) - M\ f'(\eta) = 0 \quad (8)$$

$$\theta''(\eta) + \Pr\left[f(\eta)\theta'(\eta) - \frac{A}{2}\eta\theta'(\eta) - \theta(\eta) f'(\eta) \right] = 0 \quad (9)$$

$$\phi''(\eta) + Sc\left[f(\eta)\phi'(\eta) - \frac{A}{2}\eta\phi'(\eta) - \phi(\eta) f'(\eta) \right] = 0 \quad (10)$$

$$f(0) = f_0, \quad f'(0) = \theta(0) = \phi(0) = 1$$

$$f(\infty) = \theta(\infty) = \phi(\infty) = 0 \quad (11)$$

Where primes denote differentiation with respect to similarity variable η and

$A = \dfrac{\lambda}{c}$ = Unsteadiness parameter, M = Magnetic field parameter

G_T = Thermal Grashof number G_C = Concentration Grashof number

$\Pr = \dfrac{\upsilon}{\alpha}$ = Prandlt number, $Sc = \dfrac{\upsilon}{D}$ = Schmindt number, f_0 = Injection Parameter

Results and Discussion

The set of non-linear ordinary differential equations (8)-(10) with boundary conditions in (11) have been solved numerically by using Nachtsheim-Swigert shooting iteration technique along with Runge-Kutta initial value solver with *A*, *M*, G_T, G_C, *Pr*, *Sc* and f_0 as prescribed parameters. The computations were done by a program which uses a symbolic and computational computer language MAPLE.

Here, we assigned physically realistic numerical values to the embedded parameters in the system in order to gain an insight into the flow structure with respect to velocity, temperature and concentration profiles. The results are presented in Tables 1 and graphically in Figures 1-9 and conclusions are drawn for the flow field and other physical quantity of interest that have significant effects. The effects of the flow parameters on the velocity field, temperature field and concentration field have been studied and discussed with the help of velocity profiles shown in Figures (1)-(3), temperature profiles shown in Figures (4)- (6) and concentration profiles shown in Figures (7) – (9). The effects of various parameters on skin-friction coefficient, Nusselt number and Sherwood number are shown in Table 1. The conclusions and discussion regarding the behaviour of the parameters on skin-friction coefficient, Nusselt number and Sherwood number are self-evident from the table and hence are not discussed for brevity.

Table 1: Skin-friction, the Nusselt number and the Sherwood number for the various thermo-physical parameters

A	G_T	G_C	M	Pr	Sc	f_0	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	0.1	0.1	0.1	0.72	0.62	0.1	1.13783822805	0.82371435573	0.69418382027
0.5	0.1	0.1	0.1	0.72	0.62	0.1	0.93580380230	0.87409576293	0.74531180174
1	0.1	0.1	0.1	0.72	0.62	0.1	1.05138487479	1.03994026307	0.49919958353
0.1	0.1	0.1	0.5	0.72	0.62	0.1	1.29919803602	0.78953637002	0.65951054176
0.1	0.1	0.1	1	0.72	0.62	0.1	1.47824008133	0.75325480445	0.62282460752
0.1	0.1	0.1	0.1	0.72	0.62	0.5	1.37428570969	0.98310695880	0.83176166169
0.1	0.1	0.1	0.1	0.72	0.62	1	1.71295260324	1.21924947672	1.03804841780

MAIN VELOCITY FIELD

The main velocity of the flow field suffers a change in magnitude due to the variations in the values of magnetic parameter (*M*), unsteadiness parameter (*A*) and injection parameter (f_0). The effects of these parameters appear in figures (1) - (3) respectively.

Effect of magnetic parameter (M)

The effect of the magnetic parameter on the main velocity of the flow field is shown in Fig. (1).

The presence of a magnetic field normal to the flow acts against the flow. This resistive force tends to slow down the flow and hence the fluid velocity decreases with the increase of the magnetic field parameter as observed.

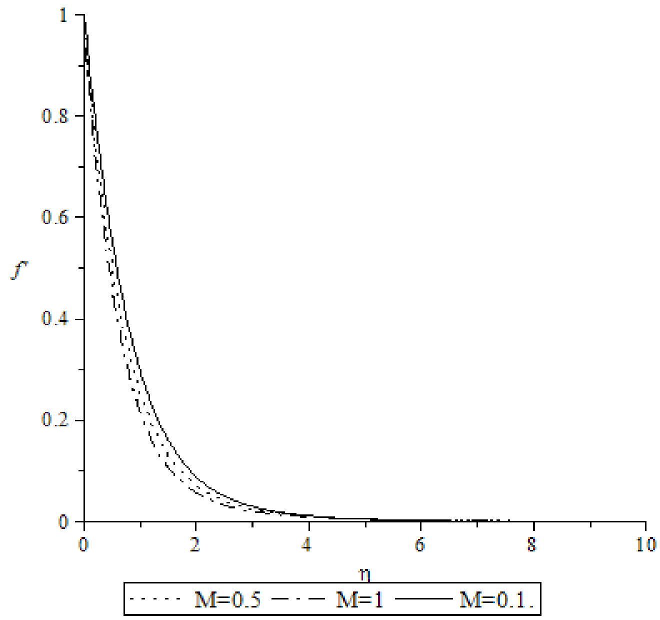

Figure 1: Velocity profiles for various values of M when $A = G_T = G_C = f_0 = 0.1$, $Pr = 0.72$, $Sc = 0.62$.

Effect of unsteadiness parameter (A)

Figure 2 shows the profiles for the velocity for various values of the unsteadiness parameter, A=0.5, 0.1 and 1, when $G_T = M = G_C = f_0 = 0.1$, $Pr = 0.72$, $Sc = 0.62$. It is evident that the increase in the unsteadiness parameter leads to an increase in the velocity.

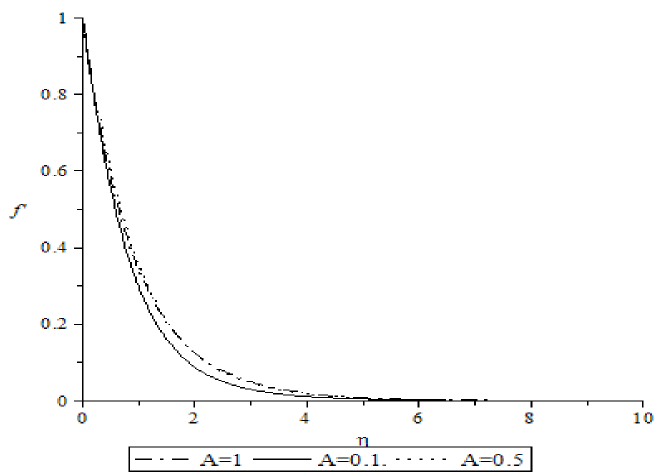

Figure 2: Velocity profiles for various values of A when $M = G_T = G_C = f_0 = 0.1$, $Sc = 0.62$, $Pr = 0.72$.

Effect of injection parameter (f_0)

The variations in fluid Velocity profiles for various values of the injection parameter (f_0), when A =M = GT = GC = $\tau = 0.1$, Pr = 0.72, Sc = 0.62, are shown in Figures 3. From Figure 3, we found out that the flow velocity decreases with the increase of injection parameter (f_0).

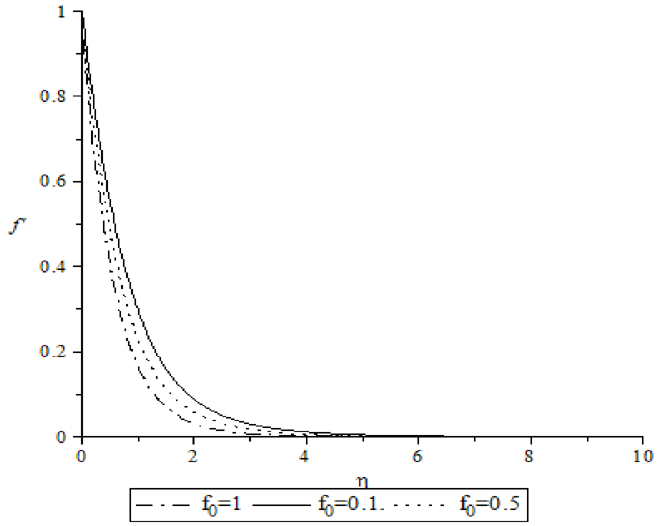

Figure 3: Velocity profiles for various values of f_0 when A =M = G_T = G_C = 0.1, Pr = 0.72, Sc = 0.62.

TEMPERATURE FIELD

The effects of the variations in the values of magnetic parameter (*M*), unsteadiness parameter (*A*) and injection parameter (f_0) on the temperature profiles of the flow is presented in figures (4) - (6) respectively.

Effect of magnetic parameter (M)

From Figure 4, we see that the temperature profiles increase with the increase of the magnetic field parameter, which implies that the applied magnetic field tends to heat the fluid, and thus reduces the heat transfer from the wall.

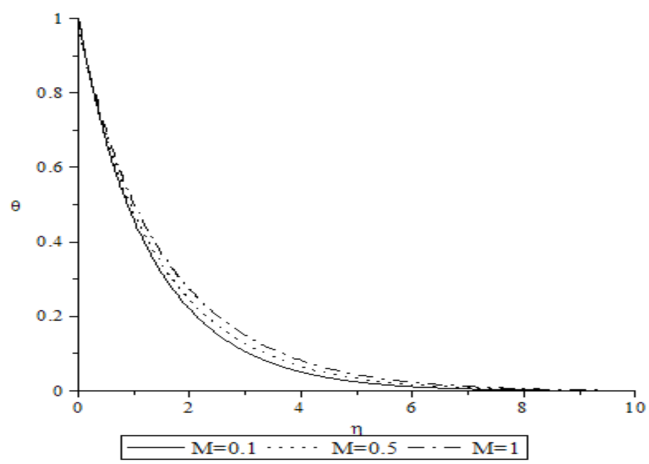

Figure 4: Temperature profiles for various values of M when A = G_T = G_C = f_0 = 0.1, Pr = 0.72, Sc = 0.62.

Effect of unsteadiness parameter (A)

Figure 5 present the curve of temperature profiles for various values of A. Increasing the value of A bring a decrease in the thermal boundary layer thickness across the plate. It is observed from that the temperature profiles decreases with the increase of the unsteadiness parameter.

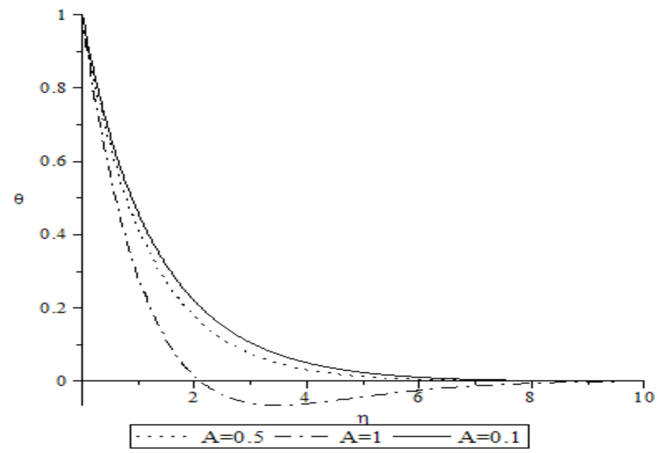

Figure 5: Temperature profiles for various values of A when $M = G_T = G_C = f_0 = 0.1$, $Sc = 0.62$, $Pr = 0.72$.

Effect of injection parameter (f_0)

Figure (6) depicts the effect of injection parameter on the temperature of the flow field. In presence of growing injection, the temperature of the flow field is found to decrease. It reveals that temperature decreases with the increase of the injection parameter.

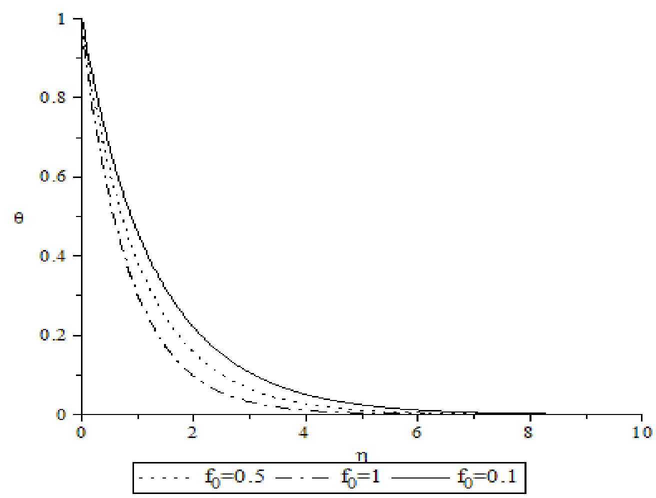

Figure 6: Temperature profiles for various values of f_0 when $A = M = G_T = G_C = 0.1$, $Pr = 0.72$, $Sc = 0.62$.

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CONCENTRATION FIELD

Figures (7) – (9), depicts the analysis of the concentration profiles of the flow as a result of the variations in the values of magnetic field parameter (*M*), unsteadiness parameter (*A*) and injection parameter (f_0).

Effect of magnetic parameter (M)

In Figure 7, we analyze the Concentration profiles for various values of M when $A = G_T = G_C = f_0 = 0.1$, $Pr = 0.72$, $Sc = 0.62$. The effect of an applied magnetic field is found to decrease the concentration profiles, and hence increase the concentration boundary layer.

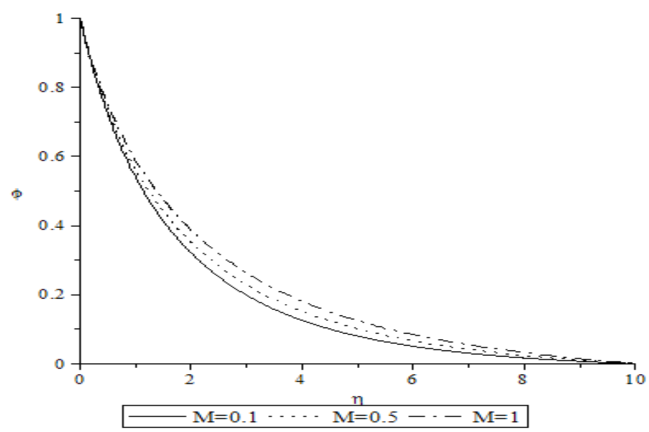

Figure 7: Concentration profiles for various values of M when $A = G_T = G_C = f_0 = 0.1$, $Pr = 0.72$, $Sc = 0.62$.

Effect of injection parameter (f_0)

In Figure 9, the effect of injection parameter on the concentration profiles for various values of f_0 when $A = M = G_T = G_C = 0.1$, $Pr = 0.72$, $Sc = 0.62$, are shown. It is seen from this figure that there is a decrease in the flow concentration with the increase of injection parameter (f_0).

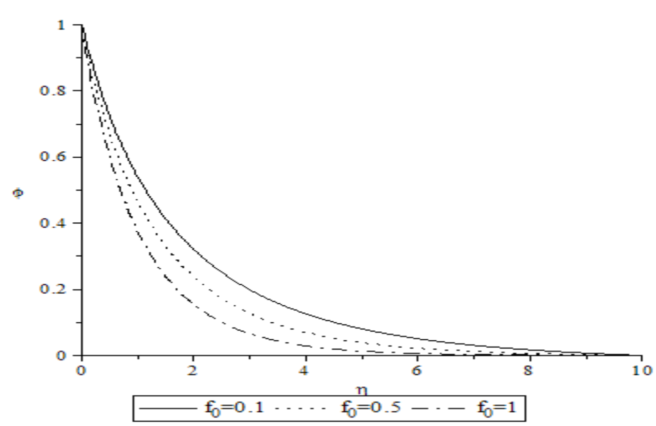

Figure 8: Concentration profiles for various values of f_0 when A =M = G_T = G_C = 0.1, Pr = 0.72, Sc = 0.62.

Conclusions

In this paper we have studied numerically the magnetic and injection effects on unsteady heat and mass laminar flow past a vertical surface. This study brings out the following interesting features of physical interest on the velocity, temperature and concentration profiles of the flow field:

a) The magnetic parameter (M) retards the main velocity (u) at all points of the flow field due to the magnetic pull of the Lorentz force acting on the flow field. The decrease of the fluid velocity is associated with a reduction in the velocity gradient at the wall, and thus the local skin-friction coefficient decreases. Also, the applied magnetic field tends to decrease the wall temperature gradient and concentration gradient, which yield a decrease the local Nusselt number and the local Sherwood number.

b) A growing injection parameter decreases the main velocity, reduces the temperature and the concentration of the flow field at all points.

c) The effect of increasing the unsteadiness parameter is that the velocity tends to rise while there is reduction in both the temperature and concentration profiles of the flow.

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